Lösungen zu den Übungsaufgaben zu Ableitungen

Ganzrationale Funktionen:

a) $f(x) = 2ax^3 - 6a^2x^2 \Rightarrow f'(x) = 6ax^2 - 12a^2x$

b) $f(x) = t^2x^4 - 3t^3x^2 + 4t^2 \Rightarrow f'(x) = 4t^2x^3 - 6t^3x$

c) $f(x) = (4x+1)^3 \Rightarrow f'(x) = 3(4x+1)^2 \cdot 4 = 12(4x+1)^2$

d) $f(x) = (2x^2+a)^4 \Rightarrow f'(x) = 4(2x^2+a)^3 \cdot 4x = 16x \cdot (2x^2+a)^3$

e) $f(x) = (x-1)\cdot(x-k)^2 \Rightarrow f'(x) = 1\cdot(x-k)^2 + (x-1)\cdot 2\cdot(x-k)\cdot 1 = (x-k)\cdot(x-k+2x-2)$
$= (x-k)\cdot(3x-k-2)$

f) $f(x) = 2ax(x-a)^2 \Rightarrow f'(x) = 2a(x-a)^2 + 2ax\cdot 2(x-a)\cdot 1 = 2a(x-a)\cdot(x-a+2x)$
$= 2a(x-a)\cdot(3x-a)$

Gebrochenrationale Funktionen:

a) $f(x) = \frac{4}{(2x+1)^2} \Rightarrow f'(x) = \frac{0-4\cdot 2(2x+1)\cdot 2}{(2x+1)^4} = \frac{-16}{(2x+1)^3}$

b) $f(x) = \frac{x}{(3x+2)^2} \Rightarrow f'(x) = \frac{1\cdot(3x+2)^2 - x\cdot 2(3x+2)\cdot 3}{(3x+2)^4} = \frac{(3x+2)-6x}{(3x+2)^3} = \frac{-3x+2}{(3x+2)^3}$

c) $f(x) = \frac{ax}{x^2+a} \Rightarrow f'(x) = \frac{a\cdot(x^2+a) - ax\cdot 2x}{(x^2+a)^2} = \frac{-ax^2+a^2}{(x^2+a)^2}$

d) $f(x) = \frac{x^3-2x^2+2}{x^2+1} \Rightarrow f'(x) = \frac{(3x^2-4x)(x^2+1)-(x^3-2x^2+2)\cdot 2x}{(x^2+1)^2} = \frac{x^4+3x^2-8x}{(x^2+1)^2}$

e) $f(x) = \frac{ax^2+2}{x^2+a} \Rightarrow f'(x) = \frac{2ax(x^2+a)-(ax^2+2)\cdot 2x}{(x^2+a)^2} = \frac{2a^2x-4x}{(x^2+a)^2}$

f) $f(x) = \frac{3x^3+2x-5}{7x^2} = \frac{3}{7}x + \frac{2}{7}x^{-1} - \frac{5}{7}x^{-2} \Rightarrow f'(x) = \frac{3}{7} - \frac{2}{7}x^{-2} + \frac{10}{7}x^{-3}$

Die Aufgabe f) könnte man auch mit der Quotientenregel ableiten.

Exponentialfunktionen:

a) $f(x) = 3x^2\cdot e^{-4x} \Rightarrow f'(x) = 6x\cdot e^{-4x} + 3x^2\cdot e^{-4x}\cdot(-4) = 6x\cdot e^{-4x}(1-2x)$

b) $f(x) = \frac{1}{2}x^3\cdot e^{2x} \Rightarrow f'(x) = \frac{3}{2}x^2\cdot e^{2x} + \frac{1}{2}x^3\cdot 2e^{2x} = x^2e^{2x}(\frac{3}{2}+x)$

c) $f(x) = (2x+5)\cdot e^{-x} \Rightarrow f'(x) = 2\cdot e^{-x} + (2x+5)\cdot e^{-x}\cdot(-1) = e^{-x}(2-2x-5) = e^{-x}(-2x-3)$

d) $f(x) = (x+k)\cdot e^{-kx} \Rightarrow f'(x) = 1\cdot e^{-kx} + (x+k)\cdot e^{-kx}\cdot(-k) = e^{-kx}(1-kx-k^2)$

e) $f(x) = (4x+e^{-x})^2 \Rightarrow f'(x) = 2(4x+e^{-x})\cdot(4-e^{-x})$

f) $f(x) = (e^x+e^{-x})^2 \Rightarrow f'(x) = 2(e^x+e^{-x})\cdot(e^x-e^{-x})$

Trigonometrische Funktionen:

a) $f(x)=2x\cdot\cos(\frac{1}{2}x^2+4)\Rightarrow f'(x)=2\cos(\frac{1}{2}x^2+4)-2x\cdot\sin(\frac{1}{2}x^2+4)\cdot x$

b) $f(x)=x^2\cdot\sin(4x+3)\Rightarrow f'(x)=2x\cdot\sin(4x+3)+x^2\cdot\cos(4x+3)\cdot 4$

c) $f(x)=(\sin x+\cos x)^2\Rightarrow f'(x)=2(\sin x+\cos x)\cdot(\cos x-\sin x)$

d) $f(x)=(x^2-\sin x)^3\Rightarrow f'(x)=3(x^2-\sin x)^2\cdot(2x-\cos x)$

e) $f(x)=(ax-\sin(ax))^2\Rightarrow f'(x)=2(ax-\sin(ax))\cdot(a-\cos(ax)\cdot a)$

f) $f(x)=x\cdot\sin(x)\cdot\cos(x)$

Nebenrechnung: $g(x)=x\cdot\sin(x)\Rightarrow g'(x)=\sin(x)+x\cdot\cos(x)$

$f(x)=g(x)\cdot\cos(x)$

$\Rightarrow f'(x)=g'(x)\cdot\cos x+g(x)\cdot(-\sin(x))=(\sin(x)+x\cdot\cos(x))\cdot\cos(x)-x\cdot\sin^2(x)$

Wurzelfunktionen:

a) $f(x)=\sqrt{x^2+4}\Rightarrow f'(x)=\frac{1}{2\sqrt{x^2+4}}\cdot 2x=\frac{x}{\sqrt{x^2+4}}$

b) $f(x)=\sqrt{ax^2-2ax}\Rightarrow f'(x)=\frac{1}{2\sqrt{ax^2-2ax}}\cdot(2ax-2a)=\frac{ax-a}{\sqrt{ax^2-2ax}}$

c) $f(x)=(\sqrt{x}+2)^2\Rightarrow f'(x)=2(\sqrt{x}+2)\cdot\frac{1}{2\sqrt{x}}=1+\frac{2}{\sqrt{x}}$

d) $f(x)=2x\sqrt{x^2+4}\Rightarrow f'(x)=2\sqrt{x^2+4}+2x\cdot\frac{1}{2\sqrt{x^2+4}}\cdot 2x=2\sqrt{x^2+4}+\frac{2x^2}{\sqrt{x^2+4}}$

e) $f(x)=x^2\sqrt{x^2-4}\Rightarrow f'(x)=2x\sqrt{x^2-4}+x^2\cdot\frac{1}{2\sqrt{x^2-4}}\cdot 2x=2x\sqrt{x^2-4}+\frac{x^3}{\sqrt{x^2-4}}$

f) $f(x)=\sqrt[5]{2x^2+5}=(2x^2+5)^{1/5}\Rightarrow f'(x)=\frac{1}{5}(2x^2+5)^{-4/5}\cdot 4x$

Logarithmusfunktionen:

a) $f(x)=\ln(2+3x^2)\Rightarrow f'(x)=\frac{6x}{2+3x^2}$

b) $f(x)=\ln(2x^2+x)\Rightarrow f'(x)=\frac{4x+1}{2x^2+x}$

c) $f(x)=2x\cdot\ln(4+x)\Rightarrow f'(x)=2\cdot\ln(4+x)+2x\cdot\frac{1}{4+x}$

d) $f(x)=\ln(x^2+t)\Rightarrow f'(x)=\frac{2x}{x^2+t}$

e) $f(x)=(x^2-2x)\ln(x^2+1)\Rightarrow f'(x)=(2x-2)\ln(x^2+1)+(x^2-2x)\cdot\frac{2x}{x^2+1}$

f) $f(x)=\ln(\ln(3x+1))\Rightarrow f'(x)=\frac{1}{\ln(3x+1)}\cdot\frac{3}{3x+1}$

Gebrochene Exponentialfunktionen:

a) $f(x)=\frac{3}{1+e^{x}}\Rightarrow f'(x)=\frac{-3\cdot e^{x}}{(1+e^{x})^{2}}$

b) $f(x)=\frac{x}{2+e^{3x}}\Rightarrow f'(x)=\frac{1(2+e^{3x})-x\cdot 3e^{3x}}{(2+e^{3x})^{2}}$

c) $f(x)=\frac{x^{2}}{1+e^{-x}}\Rightarrow f'(x)=\frac{2x(1+e^{-x})-x^{2}\cdot(-e^{-x})}{(1+e^{-x})^{2}}=\frac{x(2+2e^{-x}+x\cdot e^{-x})}{(1+e^{-x})^{2}}$

d) $f(x)=\frac{e^{x}}{2-e^{-x}}\Rightarrow f'(x)=\frac{e^{x}(2-e^{-x})-e^{x}\cdot e^{-x}}{(2-e^{-x})^{2}}=\frac{2e^{x}-2}{(2-e^{-x})^{2}}$

e) $f(x)=\frac{e^{x}+e^{-x}}{1+e^{x}}\Rightarrow f'(x)=\frac{(e^{x}-e^{-x})(1+e^{x})-(e^{x}+e^{-x})\cdot e^{x}}{(1+e^{x})^{2}}=\frac{e^{x}-e^{-x}-2}{(1+e^{x})^{2}}$

f) $f(x)=\frac{4}{1-e^{-x}}\Rightarrow f'(x)=\frac{-4e^{-x}}{(1-e^{-x})^{2}}$

Gebrochene trigonometrische, Wurzel- und ln-Funktionen:

a) $f(x)=\frac{2}{1+\sin(x)}\Rightarrow f'(x)=\frac{0-2\cdot\cos(x)}{(1+\sin(x))^{2}}$

b) $f(x)=\frac{x^{2}}{2+\cos(x)}\Rightarrow f'(x)=\frac{2x(2+\cos(x))-x^{2}\cdot(-\sin(x))}{(2+\cos(x))^{2}}=\frac{2x(2+\cos(x))+x^{2}\cdot\sin(x)}{(2+\cos(x))^{2}}$

c) $f(x)=\frac{4x}{\sqrt{1+x^{2}}}\Rightarrow f'(x)=\frac{4\sqrt{1+x^{2}}-4x\cdot\frac{2x}{2\sqrt{1+x^{2}}}}{1+x^{2}}=\frac{4(1+x^{2})-4x^{2}}{(1+x^{2})\sqrt{1+x^{2}}}=\frac{4}{(1+x^{2})\sqrt{1+x^{2}}}$

d) $f(x)=\frac{5}{\sqrt{2x+5}}\Rightarrow f'(x)=\frac{0-5\cdot\frac{2}{2\sqrt{2x+5}}}{2x+5}=\frac{-5}{(2x+5)\sqrt{2x+5}}$

e) $f(x)=\ln\left(\frac{1+x}{1-x}\right)=\ln(1+x)-\ln(1-x)\Rightarrow f'(x)=\frac{1}{1+x}-\frac{1}{1-x}\cdot(-1)=\frac{1}{1+x}+\frac{1}{1-x}$

f) $f(x)=\frac{\ln(ax)}{2x+1}\Rightarrow f'(x)=\frac{\frac{a}{ax}(2x+1)-\ln(ax)\cdot 2}{(2x+1)^{2}}=\frac{2+\frac{1}{x}-2\ln(ax)}{(2x+1)^{2}}$

Sonstige Funktionen:

a) $f(x)=\sin(e^{x}+1)\Rightarrow f'(x)=\cos(e^{x}+1)\cdot e^{x}$

b) $f(x)=e^{x}\cdot\sin(x)\Rightarrow f'(x)=e^{x}\cdot\sin(x)+e^{x}\cdot\cos(x)=e^{x}(\sin(x)+\cos(x))$

c) $f(x)=e^{3x}\cdot\sin(e^{2x})$

$\Rightarrow f'(x)=3e^{3x}\sin(e^{2x})+e^{3x}\cos(e^{2x})\cdot e^{2x}\cdot 2=e^{3x}(3\sin(e^{2x})+2e^{2x}\cos(e^{2x}))$

d) $f(x)=\sqrt{e^{3x}+1} \Rightarrow f'(x)=\dfrac{3e^{3x}}{2\sqrt{e^{3x}+1}}$

e) $f(x)=\sqrt{2x+1}\cdot e^{-x} \Rightarrow f'(x)=\dfrac{2}{2\sqrt{2x+1}}\cdot e^{-x}-\sqrt{2x+1}\cdot e^{-x}=e^{-x}\cdot(\dfrac{1}{\sqrt{2x+1}}-\sqrt{2x+1})$

f) $f(x)=\ln(\sin(x)) \Rightarrow f'(x)=\dfrac{1}{\sin(x)}\cdot\cos(x)=\dfrac{\cos(x)}{\sin(x)}$

g) $f(x)=\ln(x^2+\sin(3x)) \Rightarrow f'(x)=\dfrac{2x+3\cos(3x)}{x^2+\sin(3x)}$

h) $f(x)=\sin(\ln(3x)) \Rightarrow f'(x)=\cos(\ln(3x))\cdot\dfrac{1}{3x}\cdot 3=\dfrac{1}{x}\cos(\ln(3x))$

i) $f(x)=\sqrt{3\ln(x^2)} \Rightarrow f'(x)=\dfrac{1}{2\sqrt{3\ln(x^2)}}\cdot\dfrac{3}{x^2}\cdot 2x=\dfrac{3}{x\sqrt{3\ln(x^2)}}$

j) $f(x)=\ln\left(\sqrt{x}\right) \Rightarrow f'(x)=\dfrac{1}{\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}}=\dfrac{1}{2x}$

k) $f(x)=3\ln\left(\sqrt{2x^2+x}\right) \Rightarrow f'(x)=\dfrac{3}{\sqrt{2x^2+x}}\cdot\dfrac{4x+1}{2\sqrt{2x^2+x}}=\dfrac{12x+3}{2(2x^2+x)}$

l) $f(x)=\ln(\sin(\sqrt{e^{4x}+5})) \Rightarrow f'(x)=\dfrac{1}{\sin(\sqrt{e^{4x}+5})}\cdot\cos(\sqrt{e^{4x}+5})\cdot\dfrac{1}{2\sqrt{e^{4x}+5}}\cdot e^{4x}\cdot 4$